CREDIT PORTFOLIO SELECTION ACCORDING TO SECTORS IN RISKY ENVIRONMENTS: MARKOWITZ PRACTICE

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ABSTRACT

In this study, it was researched that how the rate of repayment of loans will be increased and how the credit risk will be minimized in banking sector, by using Markowitz Portfolio Theory. Construction, textile and wholesale and retail sectors were examined under the central bank data. Portfolio groups were selected and risks (variances of Portfolio groups) were evaluated according to Markowitz portfolio theory. Markowitz portfolio theory is effective than the other portfolio selection instruments. Although Classical risk measurement tools measure risks, but they do not be able to answer how the risks can be reduced. On the other hand, Markowitz portfolio model, which is used in this study, show how the risks can be reduced.

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1. INTRODUCTION

In banking sector, diversification of credits according to sectors are important to understand repayment ratios of loans or credit risks in detail. Credit reports on the basis of sectors show the financial performances of sectors periodically. This research is based on maximizing performing loans of banks, besides reduce the volatility of repayment -risk-. Unfortunately, banks have faced with credit defaults. Banks have faced with 3 problems that are caused by credit defaults; 1) Repayment time of defaulted loans is unknown. 2) Defaulted loans may become non-performing and it may be totally defict. 3) Legal period and costs of liquidation of default credits exceed the principles.

Banks calculate the loss rate in case of default according to Basel II standards. This loss increases the credit costs and also affects profit of banks in a negative way. In the light of foregoing, high performing rate of credits provide banks to produce stable and forward-looking
policy. Moreover this also enables to reduce liquidity crunch which is results from decrease in repayment of credits because of credit defaults. The credits provided by banks are followed from Central Bank monthly data of non-accruing loans according to sectors. It is important to examine performing loans according to distinct sectors because sectors are different in terms of their internal dynamics. For instance, The credits which is used by sector A have better performance than sector B according to performing loans however sector A can have high volatility repayment performance. This shows in some case higher performing loans rate means higher risks. The mortgage crisis rose in USA at 2008 directly affected mining industry in Turkey since mining sector’s firms mostly export their goods to the USA. This crisis caused close down the most mining firm. The other aspect of that crisis was performing loans of mining firms were sharply decreased and default credits were increased respectively. In 2008 financial global crisis the banks faced with huge losse. For instance a bank consider only sectors’ with respect to performing loans rate and disregard the volatility of the repayment performance than the bank may face with unexpected credit repayment rate because of the risks- volatility of repayment rate. Therefore it is important for banks to minimize the volatility of repayment rate and maximize performing loans so banks should determine a portfolio group of sectors to split the risk into different sectors and avoid inherent risk of sectors as mining sector mentioned above.

2. LITERATURE REVIEW

Optimizing a portfolio is a major area in finance. The aim of portfolio optimization is maximizing the income of portfolio, simultaneously minimizing the risk. One way of optimizing a portfolio was suggested by Harry Max Markowitz (born August 24, 1927) who published the article-Portofolio selection- in Journal of Finance 1952. The most important aspect of Markowitz’ model was his description of the impact on portfolio diversification by the number of securities within a portfolio and their covariance relationships (Megginson, 1996). His studies, Markowitz (1952), were first published in The Journal of Finance. Subsequently, Markowitz significantly expanded his studies in his book, Portfolio Selection: Efficient Markowitz (1959). In 1990 he received the Nobel Memorial Prize in Economic Sciences due to his contributions to portfolio theory.

In this study, our purpose is to how a bank extend credits to the sectors to aim for maximizing performing loans. In order to understand this study we need some background information about the concepts of portfolio theory. Modern Portfolio Theory is based upon Harry Max Markowitz’s studies. Before Markowitz, prices of stock exchanges had been determined by “Theory of Investment Value” which was stated by John (1938). According to Theory of Investment, it was enough to regard expected value for determining future stock value that is when an investor want to maximize the profit it was enough looking into only one stock. However, Markowitz told that risk should not be ignored in a risky environment. He associated risk with variance and came up with a new theory Markowitz (1991). Markowitz analyzed stocks under the historical datas in a risky environment and studied to minimize risks while maximize yields. In addition, Tomas et al. (2012)
analyzes the evolution of the systematic risk of the banking industries in eight advanced countries using weekly data from 1990 to 2012.

Radovan and Juraj examined to find determinants of Loss given default using a set of firm loan micro-data of an anonymous Czech commercial bank. Antoine and Bertille (2012) develop a new framework to provide a feasible optimal investment rule that accounts for estimation risk. Disatnik and Katz (2012) introduce a portfolio strategy that generates a portfolio, with no short sale positions, that can outperform the 1/N portfolio. Tu and Zhou (2011) propose an optimal combination of the naive 1/N rule with one of the four sophisticated strategies the Markowitz rule, the Jorion (1986) rule, the Mackinlay and Pastor (2000) rule, and the Kan and Zhou (2007) rule as a way to improve performance. They found that the combined rules not only have a significant impact in improving the sophisticated strategies, but also outperform the 1/N rule in most scenarios. Graham (2013) examined systematically the contributions to the existing literature by showing: (a) that the CAPM fails empirically when applied to industries; (b) that after 1993 the value and momentum anomalies appear to continue whereas the beta anomaly appears to have weakened; and (c) that the value and momentum anomalies, and the value of beta, can largely be ignored if the task is to estimate industry cost of equity. Bai et al. (2009) develop new bootstrap-corrected estimations for the optimal return and its asset allocation and prove that these bootstrap corrected estimates are proportionally consistent with their theoretic counterparts.

Alexander and Gordon (2009) first introduced value-at-risk as a measure of risk and how it relates to standard deviation, the risk measure at the heart of the model of Markowitz. Second, similarly introduced conditional value-at-risk (also known as expected shortfall) as a measure of risk and compare it with VAR. Third, they briefly introduced stress testing as a supplemental means of controlling risk. Bris et al. (2008), discussed how securities investors could protect themselves from risk through diversification. Greyserman et al. (2006) studied portfolio selection methodology using a Bayesian forecast of the distribution of returns by stochastic approximation.

Lee (1977), Litzenberger and Kraus (1976) made alternative studies about distribution of return in portfolio theory (Elton et al., 1998), without regard to developed by William Sharpe, John Lintner, and Jan Mossin, another important capital markets theory evolved as an outgrowth of Markowitz’ and Tobin’s earlier works—The Capital Asset Pricing Model (CAPM) (Megginson, 1996). The CAPM provided an important evolutionary step in the theory of capital markets equilibrium, better enabling investors to value securities as a function of systematic risk.

Sharpe (1964) significantly advanced the Efficient Frontier and Capital Market Line concepts in his derivation of the CAPM. Sharpe would later win a Nobel Prize in Economics for his seminal contributions. A year later, Lintner (1965) derived the CAPM from the perspective of a corporation issuing shares of stock. Finally, in 1966, Mossin also independently derived the CAPM, explicitly specifying quadratic utility functions (Megginson, 1996). Since the earlier works of Markowitz, and later, Sharpe, Lintner and Mossin, there have been various expansions and iterations of MPT. The remainder of this essay addresses a perceived “simplicity” gap in that literature, and suggests a
systemic failure of theorists and practitioners to capitalize upon the tremendous advances in finance and technology. It also specifically extends the conceptual premises of Wharton professor, Benniga (2006), wherein he argues for a more simplistic approach to understanding and calculating the various mathematical concepts underlying MPT.

Overall, the risk component of MPT can be measured, using various mathematical formulations, and reduced via the concept of diversification which aims to properly select a weighted collection of investment assets that together exhibit lower risk factors than investment in any individual asset or singular asset class. Diversification is, in fact, the core concept of MPT and directly relies on the conventional wisdom of “never putting all your eggs in one basket” (Fabozzi et al., 2002; Veneeya, 2006; Mcclure, 2010). It is instructive to note here that Markowitz’ portfolio selection theory is a ‘normative theory.’ Fabozzi et al. (2002) define a normative theory as “one that describes a standard or norm of behavior that investors should pursue in constructing a portfolio. Conversely, Sharpe’s asset pricing theory (CAPM) is regarded as a ‘positive theory’—one that hypothesizes how investors actually behave as opposed to how they should behave.

Together, they provide a theoretical framework for the identification and measurement of investment risk and the development of relationships between expected return and risk. There remains a degree of debate as to whether or not MPT is interdependent upon the validity of asset pricing theory (Fabozzi et al., 2002). This analysis assumes that MPT is indeed independent of asset pricing theory, with the latter concept the subject of separate analysis. Portfolio Theory basically based upon selection of best portfolio group from listed stocks. Portfolio Theory arguments could have been applied in different areas. For example; In agricultural economies, grain prices are periodic and market price of grains are changed month by month.

Because there are more than one grain types and their harvest times are different. For this reason storage problems are risen for producers who have limited storage area what kind of grains are stocked up and what their quantities should be are problems for warehouse owner. In addition, unknown future market price of stored grains pose a risk. In that case, maximizing the return of the portfolio consist of different varieties of stored grains under the risk–unknown future market price–is a problem. That problem was illustrated using quadratic equation and solved under linear constraint by using Markowitz-Portfolio arguments Heifner (1966). Todays there are legal football-betting web sites which are based on to estimate score of matches. Due to the scores which are occurred at a point of match or end of the match, bets’ return changes Match of the soccer teams are analyzed statistically using historical data. For this analyze it is accepted that the goals are reasonable for constant mean-Poisson distribution. Times of goals are not certain according to the probability so it entertains risk. In addition, mean of time intervals of goals may changes. In football betting game return rate is a negative function of mean of scoring a goal in a time interval. From this point of view, for different time intervals different bets can be played and different returns can be gained. In this position while maximizing the return the risk will increase. Thus, different forecasts for different time intervals will be selected for portfolio group. Appropriate portfolio group which optimize the return is selected according to Markowitz Theory Fitt (2009).
One of the main problems of a company is to construct its own supply chain. Companies should determine their suppliers in terms of revenues and reliability of the suppliers. If the company cannot obtain the goods which are needed to continue its production, this may cause counterparty default risk and also bankruptcy of the firm. Because a single supplier is a great risk, firms should work with alternative suppliers and make a combination of suppliers (portfolio group) to maximize the revenues which minimize risks. To sum up, the problem is formulated below:

\[ N = \text{number of suppliers} \]

\[ \mathbb{E}(R_P) = \sum_{i=1}^{n} X_i \mathbb{E}(R_i) \]  \hspace{1cm} (1)

Return on portfolio

\[ \text{Var} = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \rho_{ij} \sigma_i \sigma_j \]  \hspace{1cm} (2)

Variance of portfolio=Portfolio risk

Where \( \sigma^2 \) is variance and \( \rho \) is correlation coefficient.

And the equation (2) is minimized by using Markowitz Theory arguments by Lao and Liu (2007). The technology of generating the electrical energy is important for energy efficiency and production costs. In USA and Switzerland, the technology of electrical energy (coal, gas, nuclear, wind, oil, solar etc.) was changed in years according to production costs. The return was different for different combinations of sources. Krey and Zweifel (2008), in their studies, determined which combinations provide the optimum return for USA and Switzerland by using Markowitz mean-variance Portfolio Theory. Moreover, in Tunisia, in the study of energy generation planning for 2010-2020, the question of how the optimum portfolio from nuclear sources and fossil sources is selected, was answered by using Markowitz Portfolio Theory. (Abdelhamid et al., 2009)

3. THE PURPOSE OF THE STUDY AND METHODOLOGY

The purpose of this research is for banks to give credit to sectors which include that the banks of credits at maximum viability rate so that they create portfolios.

In this context; The Central Bank data, t. C Wholesale, Retail Trade, Construction, textile and textile products from the sectors of modern portfolio theory has been applied to the arguments of credits separated by Markowitz.

Banks' total loan volume ratio of importance since the viability of the study is to minimize the risks and maximize the vitality of credit rates. Study; "Central Bank of the Republic of Turkey law No. 1211 44 within the framework of the 3rd item by banks will be liquidated in cash and loans obtained by the declarations" had used loans to sectors related to how much using data from live (the rate of viability of loans).

A historical data calculated data is obtained on a monthly basis. The average portfolio within the historical data to calculate the variance analysis of the risks of the sector, namely \( \hat{c} \) and cast. Volatility calculated this way, the risk of each sector with the variance.
The expected value of the portfolio (vitality) \(E(R_p)\) \(= \sum_{i=1}^{n} (X_i) E(R_i)\) constituent element of a percentage of the value of each is the sum of the products of the sector in the form of expected with. In theory, the variance for each sector or entering stock portfolio (Risk) is calculated separately and then outlining the variance of a function of the portfolio.

Various elements that make up the portfolio of this function in probability \((X_i)\) portfolio-what is risk. This study of the reasons which have been used in the theory of proportions \((X_i)\) Markowitz portfolio elements be combined portfolio \(\sigma_p^2\) (portfolio risk) minimum is calculated, the value of the portfolio turnover \((X_i)\) values instead of optimal portfolio and its goal has been reached

4. THEORY AND THE MODEL

In a time series which shows the performing rate of loans for each sector percent and if \(n\) is the number of sectors in portfolio group:

- \(X_i\) is percent of sector in portfolio
- \(\sigma_i^2\) is variance of the sector
- \(\sigma_{ij}\) is covariance of sectors
- \(E(R_i)\) is expected value of sector

For the portfolio \(p\) which has \(n\) sectors, providing that \(\sigma_p^2\) is variance of portfolio and \(E(R_p)\) is expected portfolio value;

Equation 1

\[
E(R_p) = x_1 E(R_1) + x_2 E(R_2) + \ldots + x_n E(R_n)
\]

\[
\sigma_p^2 = (x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + \ldots + x_n^2 \sigma_n^2) + (x_1 x_2 \sigma_{12} + x_1 x_3 \sigma_{13} + \ldots + x_1 x_n \sigma_{1n}) + (x_2 x_1 \sigma_{21} + x_2 x_3 \sigma_{23} + \ldots + x_2 x_n \sigma_{2n}) + \ldots + (x_n x_1 \sigma_{n1} + x_n x_2 \sigma_{n2} + \ldots + x_n x_{n-1} \sigma_{nn-1})
\]

If the equation 1 is rewritten

Equation 2

\[
E(R_p) = \sum_{i=1}^{n} X_i E(R_i)
\]

\[
\sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}
\]

Because the aim of the study is minimize the risks and the variance of portfolio represents the risk the model will be as follow:
Model 1

Minimize \[ \sigma_p^2 = (x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + \cdots + x_n^2 \sigma_n^2) \]
\[ + (x_1 x_2 \sigma_{12} + x_1 x_3 \sigma_{13} + \cdots + x_1 x_n \sigma_{1n}) \]
\[ + (x_2 x_1 \sigma_{21} + x_2 x_3 \sigma_{23} + \cdots + x_2 x_n \sigma_{2n}) \]
\[ + \cdots \]
\[ + (x_n x_1 \sigma_{n1} + x_n x_2 \sigma_{n2} + \cdots + x_n x_{n-1} \sigma_{nn}) \]

Constraint 1: \[ x_1 + x_2 + \cdots + x_n = 1 \]

[In the model \( x_i \) are percent of each component in portfolio. In other words, summation of \( x_i \) is equals to 100% of portfolio. Summation of probability of \( x_i \) has to equal to 1]

Constraint 2: \[ x_1 > 0, x_2 > 0, \ldots, x_n > 0 \]

[In the model the aim is providing credits absolutely for each sector so the percentage of each sector should be greater than 0. In other words \( \forall x_i > 0 \)]

If the Model 1 is rewritten in more complex form;

Model 2

Minimize \[ \sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \]
\[ \sum_{i=1}^{n} x_i = 1 \quad 2) \quad x_i > 0 \quad i = 1, 2, \ldots, n \]

The optimization problem is solved under constraints.

5) Application and solution of the problem

\[ Z = \text{Construction} \]
\[ Y = \text{Textile} \]
\[ X = \text{Wholesale and Retail} \]

If \( \sigma_X^2, \sigma_Y^2 \) and \( \sigma_Z^2 \) are variances of sectors respectively, \( \sigma_{xy}, \sigma_{xz} \) ve \( \sigma_{yz} \) are covariances between sectors;

\[ \sigma_X^2 = 12574.10^{-8}, \]
\[ \sigma_Y^2 = 12400.10^{-8}, \]
\[ \sigma_Z^2 = 4696.10^{-8}, \]
\[ \sigma_{xy} = 6578.10^{-8}, \]
\[ \sigma_{xz} = 3656.10^{-8}, \]
\[ \sigma_{yz} = 11818.10^{-8} \]

When the values are put into the model

\[ \sigma_p^2(x, y, z) = 12574.10^{-8}x^2 + 12400y^2 + 4696.10^{-8}z^2 \]
\[ + 6578.10^{-8}xy + 3656.10^{-8}xz + 11818.10^{-8}yz \]

Variance of portfolio X, Y and Z will be as shown at (1).

With quadratic programming variance of portfolio will be minimized effectively. The model is a minimization problem under 2 constraints which is quadratic programming model mainly.

Minimize

\[ \sigma_p^2(x, y, z) = 12574.10^{-8}x^2 + 12400y^2 + 4696.10^{-8}z^2 + 6578.10^{-8}xy + \]

\[ + 3656.10^{-8}xz + 11818.10^{-8}yz \]
Constraint 1 \( x + y + z = 1 \)
Constraint 2 \((x) > 0; (y) > 0; (z) > 0;\)
Wolframalpha web site is used for the solution of problem under constraints.¹

<table>
<thead>
<tr>
<th>Table-1. Outputs of solution of the problem</th>
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<tbody>
<tr>
<td>Function</td>
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<tr>
<td>Domain</td>
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</table>

Global minimum : Not founded.
Local minimum :

**Situation 1**

\[
\min \left\{ \frac{12574x^2}{10^8} + \frac{12400y^2}{10^8} + \frac{4696z^2}{10^8} + \frac{6578xy}{10^8} + \frac{3656xz}{10^8} + \frac{11818yz}{10^8} \right\}.
\]

\[
x + y + z = 1 \ \\ Ax > 0 \ \\ Ay > 0 \ \\ Az > 0 \} \approx 4092 \times 10^{-8} \ at (x, y, z)
\]

\[
\approx (210.66.10^{-3}, \ 0 ,789,33.10^{-3})
\]

**Situation 2**

\[
\min \left\{ \frac{12574x^2}{10^8} + \frac{12400y^2}{10^8} + \frac{4696z^2}{10^8} + \frac{6578xy}{10^8} + \frac{3656xz}{10^8} + \frac{11818yz}{10^8} \right\}.
\]

\[
x + y + z = 1 \ \\ Ax > 0 \ \\ Ay > 0 \ \\ Az > 0 \} \approx 4153 \times 10^{-8} \ at (x, y, z)
\]

\[
\approx (2162.9.10^{-4},228.8.10^{-4} ,7608,2.10^{-4})
\]

**Situation 3**

\[
\min \left\{ \frac{12574x^2}{10^8} + \frac{12400y^2}{10^8} + \frac{4696z^2}{10^8} + \frac{6578xy}{10^8} + \frac{3656xz}{10^8} + \frac{11818yz}{10^8} \right\}.
\]

\[
x + y + z = 1 \ \\ Ax > 0 \ \\ Ay > 0 \ \\ Az > 0 \} \approx 4174,9 \times 10^{-8} \ at (x, y, z)
\]

\[
\approx (2186,4.10^{-4},305,3.10^{-4} ,7508,2.10^{-4})
\]

6. DISCUSSIONS AND COMMENTS

According to the results the risk is minimum for the situation 1. When the situation 1 is analyzed because the requirement (constraints of problem) of percent of each sector in portfolio should be greater than 0 cannot be provided there is no solution. For the situation 2; percentage of wholesale sector in portfolio should be 22, textile sector should be 2%, and construction sector

¹http://www.wolframalpha.com/
should be 76% in the portfolio. When we apply the outputs into the data of November 2009 the total credits of 98,127,182,80 TL should be distributed as 21,587,980,20 TL for wholesale and retail sector, 1,962,543,70 TL for textile sector and 74,576,658,90 TL for instruction industry. When we apply the November 2009 sectoral performing rates to the portfolio group which is shown at satiation 2 total performing loans will 93,642,770,54 TL. In accordance with the performing loans 91,922,762,90 TL for November 2009 the performing rate was increased from 93,6% to 95,4%.

For situation 3 the performing rate is 95,3%. When situation 2 and 3 is analyzed variance of portfolio is $4155 \times 10^{-8}$ and $4174 \times 10^{-8}$ accordingly. In the theory the risk is determined by variance so situation 2 will be selected where the risk is minimum. In other words, for both situation 2 and 3, there is a solution, is it observed that the performing rates increased and the risk rates decreased. The situation where the $x$ is minimum is selected. As it is seen at the application portfolio theory is an instrument which decrease the risks besides measuring the risks. However, distribution of total credits to portfolio groups according to theory is not possible because of portfolio groups own credit volume. In an example, according to the data of November 2009 total credits of instruction sector was 29963716,9 however in the theory 74576658,90 tl credit should be provided which is not possible technically. Although the theory is seen as impossible to applied because of internal credit volumes of portfolio groups, it is important that the theory showed the most effective sectors for investment. Additionally the data are total credits which were provided by all financial institutions in Turkey. By this view, when the theory is applied by a bank the credits distributed to portfolio groups will not exceed the internal credit volume of each sector.

**Figur-1. Repayment of Loan in Level Month (%)**

In graph 1 wholesale, instruction and textile and performing rates of optimum portfolio of these sectors in a time interval (35 months) are shown. Optimum portfolio contains 22% of wholesale-retail sector, 2% textile sector, 76% intrusion sector. The performing rate of portfolio, which is existed by these rates, is calculated as multiplication of monthly performing rates of wholesale-retail,textile and instruction sectors with percentages. As it is seen in graph 1the
performing rate of portfolio is greater than textile and wholesale-retail sectors. Although performing rate of instruction sector is greater than performing rate of portfolio, when the reluctancy of instruction sector is \( Z = \text{INSTRUCTION} Y = \text{TEXTILE} \quad X = \text{WHOLESALE AND RETAIL} \quad \text{P} = \text{PORTFOLIO} \) \( \sigma^2_Z = 4696. \, 10^{-8} \), reluctancy of portfolio is \( \sigma^2_P = 4155. \, 10^{-8} \). In other words instruction sector is riskier than portfolio on its own.

![Figur-2. Repayment Risk](image)

(In the figure, performing percentage is showed on the vertical axis and risk is showed on the horizontal axis)

It is shown that the reluctancy of the portfolio is smaller than reluctance of each sector in portfolio. \( (\sigma^2_Z = 12574. \, 10^{-8}, \sigma^2_Y = 12400. \, 10^{-8}, \sigma^2_X = 4696. \, 10^{-8}) \) the values show portfolio is less riskier than each sector separately.

**REFERENCES**


John, B.W., 1938. The theory of investment value. Publisher Mas.


