MONETARY POLICY AND THE BEHAVIOR OF A MONOPOLISTIC BANK: A THEORETICAL APPROACH

Eleni Dalla
Department of Economics University of Macedonia Egnatia St., Thessaloniki, Greece

Erotokritos Varelas
Department of Economics University of Macedonia Egnatia St., Thessaloniki, Greece

ABSTRACT
This paper investigates the influence of monetary policy on the optimal behavior of a monopolistic bank. More specifically, we discuss how the overdraft rate and the minimum reserve requirements affect the equilibrium values of lending rate and deposit rate as well as the corresponding quantities, when there is only one commercial bank in the economy and the Central Bank. Moreover, we examine the impact of these changes on the magnitude of the spread between the equilibrium rates. It is demonstrated that monetary policy via the overdraft rate does not affect the spread, while the effect of a change in the fraction of the minimum reserve requirements differs depending on the case.

Keywords: Bank Behavior, Monetary Policy, Overdraft Rate, Reserve Requirements.

JEL Classification: C61, D42, E52, G21, L12

INTRODUCTION

The effect of monetary policy on the bank behavior is a common issue in economic literature. The Klein-Monti model (Klein, 1971) constitutes a prototype model of Industrial Organization of Banking that examines the impact of changes in the interbank rate on the behavior of a monopolistic bank. The Klein-Monti model is described and compared to alternative models of banking in the surveys of Baltensperger (1980) and Santomero (1984). Moreover, it has been generalized and extended by many authors, as for example Dermine (1986), Toolsema and Schoonbeek (1999) and Khemraj (2010). Varelas (2000) investigates the profit maximization problem of a representative commercial bank and the related comparative statics results. The author shows that both equilibrium rates on loans and deposits are positively related to the minimum reserve requirements. Karpetis et al. (2004) and (Karpeti and Varelas, 2005) follow a similar analysis to examine the effect of operational cost on bank’s profitability. The relation between
monetary policy and bank behavior has also been the object of studies that concern with oligopolistic bank sectors, as for example Smith (2002), Melnik et al. (2005), Beck et al. (2003), Glocker and Towbin (2012), Hillinger (2008).

We concentrate on the way the minimum reserve requirements and the overdraft rate affect the optimal monopolistic bank behavior. In particular, we are interested in the influence of these instruments of monetary policy on the interest rate spread. To achieve this, we extend the analysis of Varelas (2000), introducing the net position of the monopolistic bank on the model. In order to examine the effects of monetary policy on the optimal quantities and rates of both deposits and loans, comparative statics is implemented. We conclude that the minimum reserve requirements constitute the only effective policy instrument in this context.

The structure of the paper has as follows. Section 2 presents the theoretical model and its solution. Section 3 examines the implications of monetary policy. Section 4 concludes.

THE THEORETICAL MODEL

Assume there is only one commercial bank in the economy and the Central bank. The commercial bank operates as a monopolist of the economy. In order to maximize its profits subjected to the balance sheet constraint, the bank chooses the optimal rates on loans and deposits.

The demand function for loans is given by:

\[ L(r_L) = a_0 - a_1 r_L, \quad a_0 \geq 0, \quad a_1 > 0 \]  \hspace{1cm} (1)

According to relation (1), the demand for loans is a negative function of the lending rate \( r_L \).

The supply function of deposits has as follows:

\[ D(r_D) = d_0 + d_1 r_D, \quad d_0 \geq 0, \quad d_1 > 0 \]  \hspace{1cm} (2)

This function has upward slope, showing that the deposit supply and the corresponding interest rate \( r_D \) are related positively.

The difference between the volume of deposits \( D \) and the sum of the volume of loans \( L \) and the reserve requirements is defined as the net position of the bank. In the context of a monopolistic banking system, the net position cannot be positive, as in this case the bank cannot lend it on the Central Bank. Consequently, it is negative or zero. If it is negative, the bank borrows from the Central Bank to satisfy the liquidity needs. As the supreme bank of the country and the bankers’ bank, the Central Bank acts as the lender of the last resort. For this reason, the commercial bank has an overdraft account to the Central Bank.
Under the assumption of a linear functional form, the net position of the bank with respect to Central Bank is given by:

\[ M = (1 - \alpha)D - L \leq 0 \]  

(3)

where \( \alpha \in (0,1) \) denotes the fraction of the minimum reserve requirements, which is determined exogenously by the Government or the Central Bank.

The profit function of the monopolistic bank is given by the difference between total revenues (\( TR \)) and total cost (\( TC \)). That is:

\[ \Pi = TR - TC \]  

(4)

The total revenues are comprised by the total amount of the interest rate received by loans (\( r_L L \)) and the total amount of the exogenous rate of government bonds (\( r_B B \)). Moreover, the total cost is the sum of the fixed cost (\( c \)) and the variable cost. The latter has three components: the total amount of the interest rate paid to depositors (\( r_D D \)), the total amount of the overdraft rate paid to Central Bank (\( r|\widetilde{M}| \)) and the operational cost. We assume that the operational cost is a constant fraction of deposits (\( kD, 0 < k < 1 \))

Taking into account the above clarifications, the bank’s profit function is transformed as follows:

\[ \Pi(r_L, r_D) = r_L L + r_B B - r|\widetilde{M}| - r_D D - c - kD \]  

(5)

Substituting the relations (1), (2) & (3) to (5), we obtain:

\[ \Pi(r_L, r_D) = r_L(a_0 - a_1 r_L + a_1 r) + r_B + r_D [r(1 - a)d_4 - d_0 - d_1 r_D - kd_0] + r(1 - a)d_0 - ra_0 - c - kd_0 \]  

(6)

The monopolistic bank maximizes its profit function subjected to the balance sheet constrain:

\[ R + B + L = K + D \]  

(7)

where

\[ R = \alpha D, \quad 0 < \alpha < 1 \]  

(8)

The substitution of the relations (1), (2) & (8) to (7), implies:

\[ R + B + L = K + D \Rightarrow aD + B + L = K + D \Rightarrow L = K + (1 - \alpha)D - B \]  

(9)
The problem of the monopolistic bank can be stated as follows (relations (6) and (9)):

$$\max_{r_L, r_D} \Pi (r_L, r_D) = r_L (a_0 - a_1 r_L + a_3 r) + r_B B + r_D \left[ r(1 - a) d_1 - d_0 - d_1 r_D - k d_1 \right] + r(1 - a) d_0 - r a_0 - c - k d_0$$

s.t. \( a_0 - a_1 r_L = K + (1 - a)(d_0 + d_1 r_D) - B \)

In order to solve the above problem, we are going to use the Lagrangian function:

$$Q (r_L, r_D, q) = r_L (a_0 - a_1 r_L + a_3 r) + r_B B + r_D \left[ r(1 - a) d_1 - d_0 - d_1 r_D - k d_1 \right] + r(1 - a) d_0 - r a_0 - c - k d_0 + q \left[ a_0 - a_1 r_L - K - (1 - a)(d_0 + d_1 r_D) + B \right]$$

where \( q \) denotes the Lagrange multiplier. It can be interpreted as the change in the profit function due to a unit change in the bank's net position. (Proof See Appendix)

The first order necessary and sufficient conditions for an extremum are described by the following equations:

$$\frac{\partial Q(.)}{\partial r_L} = 0 \Rightarrow a_0 - 2 a_1 r_L + a_3 r - q a_1 = 0 \tag{10}$$

$$\frac{\partial Q(.)}{\partial r_D} = 0 \Rightarrow r(1 - a) d_1 - d_0 - 2 d_1 r_D - k d_1 - q(1 - a) d_1 = 0 \tag{11}$$

$$\frac{\partial Q(.)}{\partial q} = 0 \Rightarrow a_0 - a_1 r_L - K - (1 - a)(d_0 + d_1 r_D) + B = 0 \tag{12}$$

From the solution of the system of the first order conditions, we deduce the optimal rates on loans and deposits and the Lagrange multiplier, \( r_L^* (r, a, k, B, K), r_D^* (r, a, k, B, K) \) & \( q^* (r, a, k, B, K) \) respectively.

To check for maximum, we use the determinant of the bordered Hessian matrix:

$$|\tilde{H}| = \begin{vmatrix} 0 & -a_1 & -(1 - a) d_1 \\ -a_1 & -2 a_1 & 0 \\ -(1 - a) d_1 & 0 & -2 d_1 \end{vmatrix} = 2 a_1 d_1 [(1 - a)^2 d_1 + a_1] > 0$$

Consequently, the second order condition is satisfied.
MONETARY POLICY IMPLICATIONS

Taking the total differential of the first order conditions (relations (10), (11) & (12)) and presuming that $dk = dB = dK = 0$, we obtain the following system of equations in matrix form:

\[
\Delta \begin{bmatrix}
   dr_f^* \\
   dr_p^* \\
   dq^*
\end{bmatrix} = \begin{bmatrix}
   -a_1 dr \\
   -(1-a)d_1 dr + (r-q)d_1 da \\
   -(d_0 + d_1 r_p^*) da
\end{bmatrix}
\]

(13)

where $\Delta = \begin{bmatrix}
   -2a_1 & 0 & -a_1 \\
   0 & -2d_1 & -(1-a)d_1 \\
   -a_1 & -(1-a)d_1 & 0
\end{bmatrix}$

The determinant of the matrix $\Delta$, that is $|\Delta|$, is positive:

\[
|\Delta| = \begin{vmatrix}
   -2a_1 & 0 & -a_1 \\
   0 & -2d_1 & -(1-a)d_1 \\
   -a_1 & -(1-a)d_1 & 0
\end{vmatrix} =
\]

\[
= 2a_1 d_1 [a_1 + (1-a)^2 d_1] > 0
\]

(14)

The Overdraft Rate as a Policy Instrument

Assuming that $dr \neq 0$ and $da = 0$ and applying the Cramer’s Rule, we can determine the partial derivatives $r_f^*$ and $r_p^*$ with respect to $r$:

\[
\frac{\partial r_f^*}{\partial r} = 0
\]

(15)

\[&\]

\[
\frac{\partial r_p^*}{\partial r} = 0
\]

(16)

According to equations (15) and (16), a change in the overdraft rate ($r$) has no impact on the optimal interest rates on loans and deposits, respectively. Hence, the same holds in the case of the interest rate spread. That is:

\[
\frac{\partial (r_f^* - r_p^*)}{\partial r} = \frac{\partial r_f^*}{\partial r} - \frac{\partial r_p^*}{\partial r} = 0
\]

(17)

From equations (1) and (15), we find:

\[
\frac{\partial L^*}{\partial r} = \frac{\partial L^*}{\partial r_f^*} \frac{\partial r_f^*}{\partial r} = -a_1 \frac{\partial r_f^*}{\partial r} = 0
\]

(18)
Relations (2) and (16), imply:

\[
\frac{\partial D^*}{\partial r} = \frac{\partial D^*}{\partial r_d} \frac{\partial r_d^*}{\partial r} = d_1 \frac{\partial r_d^*}{\partial r} = 0 \tag{19}
\]

We infer that monetary policy via the overdraft rate affects neither the optimal interest rates on loans and deposits nor the corresponding quantities. The absence of overdraft-rate influence on the interest rates spread of the monopolistic bank is something that should be expected. The reason is that the different timing structure between deposits and lending induces in general a commercial bank to resort often to its overdraft account with the Central Bank. Consequently, changes in the overdraft rate do not affect the interest rates. That is, a commercial bank is impelled by the circumstances to internalize this short-term cost, without passing it over to its clients, since there is no interbank market as an alternative solution.

**The Minimum Reserve Requirements as a Policy Instrument**

Providing that \( dr=0 \) and \( da \neq 0 \) and applying the Cramer’s Rule, we deduce the partial derivatives of \( r^*_L \) and \( r^*_D \) with respect to \( a \):

\[
\frac{\partial r^*_L}{\partial a} = \frac{(r - q)(1 - a)d_1 + 2(d_0 + d_1r_D)}{2[a_1 + (1 - a)^2d_1]} \tag{20}
\]

\&

\[
\frac{\partial r^*_D}{\partial a} = \frac{-(r - q)a_1 + 2(1 - a)(d_0 + d_1r_D)}{2[a_1 + (1 - a)^2d_1]} \tag{21}
\]

From relations (20) and (21):

\[
\frac{\partial (r^*_L - r^*_D)}{\partial a} = \frac{\partial r^*_L}{\partial a} - \frac{\partial r^*_D}{\partial a} = \frac{(r - q)[(1 - a)d_1 + a_1] + 2a(d_0 + d_1r_D)}{2[a_1 + (1 - a)^2d_1]} \tag{22}
\]

Taking from relation (1) the partial derivative of \( L \) with respect to \( a \) and using relation (20), we obtain:

\[
\frac{\partial L^*}{\partial a} = \frac{\partial L^*}{\partial r_L} \frac{\partial r_L^*}{\partial a} = -a_1 \frac{\partial r_L^*}{\partial a} \tag{23}
\]

Similarly, from relations (2) and (21):

\[
\frac{\partial D^*}{\partial a} = \frac{\partial D^*}{\partial r_D} \frac{\partial r_D^*}{\partial a} = d_1 \frac{\partial r_D^*}{\partial a} \tag{24}
\]
It is clear that the effects of a change in \( a \) on \( L^* \) and \( r^*_L \) are of the opposite sign, while \( D^* \) and \( r^*_D \) move towards the same direction after a change in \( a \).

In order to determine the sign of the implication of monetary policy via the minimum reserve requirements on the optimal rates and amounts of loans and deposits, we apply mathematical investigation. It is noteworthy that all the interest rates, the overdraft rate, the lending rate and the deposit rate, are nominal rates and as a result their values belong in the interval \((0, 1)\).

Setting the relation (20) equal to zero and solving with respect to \( r - q \), we have:

\[
\frac{\partial r^*_L}{\partial a} = \frac{(r - q)(1 - a)d_1 + 2(d_0 + d_1r_D)}{2[a_1 + (1 - a)^2d_1]} = 0 \Rightarrow
\]

\[
\Rightarrow r - q = -\frac{2(d_0 + d_1r_D)}{(1 - a)d_1} < 0 \quad (25)
\]

Equating relation (21) to zero, we obtain:

\[
\frac{\partial r^*_D}{\partial a} = \frac{-(r - q)a_1 + 2(1 - a)(d_0 + d_1r_D)}{2[a_1 + (1 - a)^2d_1]} = 0 \Rightarrow
\]

\[
\Rightarrow r - q = \frac{2(1 - a)(d_0 + d_1r_D)}{a_1} > 0 \quad (26)
\]

Similarly, from equation (22), we have:

\[
\frac{\partial (r^*_L - r^*_D)}{\partial a} = \frac{(r - q)[(1 - a)d_1 + a_1] + 2a(d_0 + d_1r_D)}{2[a_1 + (1 - a)^2d_1]} = 0 \Rightarrow
\]

\[
\Rightarrow r - q = -\frac{2a(d_0 + d_1r_D)}{[(1 - a)d_1 + a_1]} < 0 \quad (27)
\]

The determination of the sign requires the ordering of the roots (25) and (27). From model’s assumptions, it holds that \(0 < a < 1\). So,

\[
a < 1 \Rightarrow 2a(d_0 + d_1r_D) < 2(d_0 + d_1r_D) \quad (28)
\]

Moreover, due to the fact that \( a_1 > 0 \), we get:

\[
a_1 > 0 \Rightarrow (1 - a)d_1 + a_1 > (1 - a)d_1 \Rightarrow \frac{1}{(1 - a)d_1 + a_1} < \frac{1}{(1 - a)d_1} \quad (29)
\]
Multiplying the inequalities (28) & (29) by members:

\[
\frac{2a(d_0 + d_1r_D)}{(1-a)d_1 + a_1} < \frac{2(d_0 + d_1r_D)}{(1-a)d_1} \Rightarrow -\frac{2a(d_0 + d_1r_D)}{(1-a)d_1 + a_1} > -\frac{2(d_0 + d_1r_D)}{(1-a)d_1}
\]  

(30)

The following table summarizes the sign of the impact of a change in θ on the equilibrium rates and quantities, when the term \( r - q \) is negative (Table 1):

Table 1. Determination of the effects of a change in θ, when \( r - q < 0 \)

<table>
<thead>
<tr>
<th>( r - q )</th>
<th>(-\infty, -\frac{2(d_0 + d_1r_D)}{(1-a)d_1})</th>
<th>(-\frac{2(d_0 + d_1r_D)}{(1-a)d_1}, -\frac{2a(d_0 + d_1r_D)}{(1-a)d_1 + a_1})</th>
<th>(-\frac{2a(d_0 + d_1r_D)}{(1-a)d_1 + a_1}, 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial r_L^*}{\partial \alpha} )</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \frac{\partial r_D^*}{\partial \alpha} )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \frac{\partial (r_L^* - r_D^*)}{\partial \alpha} )</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( \frac{\partial L^*}{\partial \alpha} )</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \frac{\partial D^*}{\partial \alpha} )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

It is inferred that:

- If the value interval of \( r - q \) is the first one, the optimal lending rate (\( r_L^* \)) decreases after an increase in the minimum reserve requirements, while the optimal deposit rate (\( r_D^* \)) increases. Consequently, the magnitude of the spread (\( r_L^* - r_D^* \)) declines. As far as the equilibrium levels of loans and deposits are concerned, an increase in \( \alpha \) leads in an increase in both of them.

- When \( r - q \) takes values in the second interval, a restrictive monetary policy via \( \alpha \) is followed by an increase in the optimal rate on loans (\( r_L^* \)). The same holds in the case of the equilibrium rate on deposits (\( r_D^* \)). However, the spread (\( r_L^* - r_D^* \)) declines, showing that the change in \( r_D^* \) is greater than the corresponding change in \( r_L^* \). Concerning the equilibrium amounts of loans and deposits, we can observe a reduction in the equilibrium level of loans (\( L^* \)) and a rise in the equilibrium level of deposits (\( D^* \)).

- If the value interval of \( r - q \) is the last one, an increase in \( \alpha \) leads to an increase in both the optimal rate on loans (\( r_L^* \)) and the optimal rate on deposits (\( r_D^* \)). Furthermore, the restrictive monetary policy affects positive the spread (\( r_L^* - r_D^* \)), implying that the aforementioned change in \( r_L^* \) is greater than the corresponding change in \( r_D^* \). Finally, regarding the equilibrium levels of loans and deposits, we observe a decrease in the level of loans (\( L^* \)) and an increase in the level of deposits (\( D^* \)).
Table 2 depicts the sign of the effect of a change in $\alpha$ on the optimal rates and amounts of loans and deposits, for positive values of the term $r - q$:

<table>
<thead>
<tr>
<th>$r - q$</th>
<th>$0, \frac{2(1 - \alpha)(d_0 + d_1 r_D)}{a_1}$</th>
<th>$\left( \frac{2(1 - \alpha)(d_0 + d_1 r_D)}{a_1}, +\infty \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial r_L^*}{\partial \alpha}$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\frac{\partial r_D^*}{\partial \alpha}$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\partial (r_L^* - r_D^*)}{\partial \alpha}$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\frac{\partial \Delta^<em>}{\partial L^</em>}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\partial \Delta^*}{\partial a}$</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

We observe that:

- When the value interval of $r - q$ is the first one, an increase in the fraction of reserve requirements is followed by an increase in both the optimal lending rate ($r_L^*$) and the optimal deposit rate ($r_D^*$). Furthermore, the increase in $\alpha$ leads to an increase in the spread ($r_L^* - r_D^*$), implying that the change in $r_L^*$ is greater than the corresponding change in $r_D^*$. Concerning the equilibrium amounts of loans and deposits, we can observe a reduction in the equilibrium level of loans ($L^*$) and an increase in the equilibrium level of deposits ($D^*$).

- If the value interval of $r - q$ is the second one, restrictive monetary policy via $\alpha$ leads to an increase in the equilibrium lending rate ($r_L^*$), while the deposit rate declines ($r_D^*$). Consequently, the magnitude of the spread ($r_L^* - r_D^*$) increases. Regarding the equilibrium levels of loans and deposits, both of them decrease after an increase in $\alpha$.

CONCLUSION

In this paper we examined the way the optimal bank behavior is affected by the minimum reserve requirements and the overdraft rate under monopolistic conditions. Firstly, we specified the demand function for loans and the supply function of deposits. Then, we solved the maximization problem of the monopolistic bank. Applying comparative statics analysis, we showed the effects of monetary policy on the optimal interest rates of deposits and loans and on the corresponding quantities.

We concentrated on the change that is induced on the spread between the equilibrium rates on loans and deposits by a change in the fraction of minimum reserve requirements and the overdraft rate. We demonstrated that monetary policy via the overdraft rate has no impact on the spread, while the
effect of changes in the fraction of minimum reserve requirements depends on the value of the difference between the overdraft rate and the Lagrange multiplier. Finally, in this model as well as in other contributions in the literature, for instance Hillinger (2008), an increase in the reserve requirements creates a countercyclical effect, as the volume of loans decreases in most of the cases. We see that only the first column of table 1 shows an increase in the amount of loans.

Appendix

Determination of Economic Interpretation of Lagrange Multiplier

The bank’s profit maximization problem has the following general mathematical form:

\[
\max_{r_L, r_D} \Pi(r_L, r_D) \\
\text{s.t. } R + B + L = K + D
\]

From relations (3) & (8), the above balance sheet constraint can be written as:

\[
B - K = (1 - a)D - L = M (1), (2)
\]

\[
B - K = (1 - a)(d_0 + d_1 r_D) - (a_0 - a_1 r_L) = M
\]

The solution of the maximization problem requires the formation of the Lagrange function:

\[
Q(r_L, r_D, q) = \Pi(r_L, r_D) + q[B - K - (1 - a)(d_0 + d_1 r_D) + (a_0 - a_1 r_L)]
\]

The first order conditions for an extremum have as follows:

\[
\frac{\partial Q(.)}{\partial r_L} = 0 \Rightarrow \frac{\partial \Pi(.)}{\partial r_L} - qa_1 = 0 \Rightarrow \frac{\partial \Pi(.)}{\partial r_L} = qa_1 \quad (A.1)
\]

\[
\frac{\partial Q(.)}{\partial r_D} = 0 \Rightarrow \frac{\partial \Pi(.)}{\partial r_D} - q(1 - a)d_1 = 0 \Rightarrow \frac{\partial \Pi(.)}{\partial r_D} = q(1 - a)d_1 \quad (A.2)
\]

\[
\frac{\partial Q(.)}{\partial q} = 0 \Rightarrow B - K - (1 - a)(d_0 + d_1 r_D) + (a_0 - a_1 r_L) = 0 \quad (A.3)
\]

Calculating the total differential of the objective function (profit function), we obtain:

\[
d\Pi = \frac{\partial \Pi(.)}{\partial r_L} dr_L + \frac{\partial \Pi(.)}{\partial r_D} dr_D \quad (A.1), (A.2)
\]

\[
d\Pi = qa_1 dr_L + q(1 - a)d_1 dr_D \Rightarrow
\]
\[ d\Pi = q(a_1 d\tau_L + (1 - a) d\tau_d) \quad (A.4) \]

Similarly, the total differential of balance sheet constraint implies:

\[ dB - dK = (1 - a) d\tau_d + a_1 d\tau_L = dM \quad (A.5) \]

Combining the relations \((A.4)\) & \((A.5)\), it is deduced that:

\[ d\Pi = q(dB - dK) = qdM \Rightarrow q = \frac{d\Pi}{dM} \quad (A.6) \]

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